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A Modified Model for Dust Devil Heights and Wind speeds

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Abstract

A modification of the published heat engine theory of Renno et al. [11] is suggested to allow an improved prediction of vortex height and consequent thermodynamic efficiency and wind-speeds in dust devils, through a reconsideration of the nature of the cold reservoir of the heat engine.

Introduction

Small dust-devils (dry buoyancy vortices wherein the buoyancy driving the flows is entirely inside the core) are a common phenomenon in deserts. Large tall dust-devils are rare, but can produce high wind-speeds at their bases. The objective of this work is to improve the prediction of vortex height and associated core temperature, so that the Carnot efficiency can be found.

Aspects of Buoyancy Vortices

Structure

A buoyancy vortex can be considered as containing four zones:

- Buoyant core:
- Potential vortex: a larger cylinder around the core, containing weaker cyclostrophic flows.
- End wall disc: the area where the potential flows meet the ground and are subject to the end-wall effect.
- Plume above: the turbulent plume formed down-wind of (above) the core when core restraint breaks down.

Processes

Three processes are fundamental in a buoyancy vortex:

- End Wall Effect
- Heat Engine
- Hydrodynamic Stability

The persistence of the vortex to altitude depends upon the hydrodynamic stability of the core wall suppressing turbulent mixing. There is still diffusion of vorticity outwards to the potential vortex around it as the core advects so it ‘winds down’ with increasing height until the wall can no longer suppress turbulence - at which point the core breaks down to a turbulent plume - unless some other process can overcome the diffusion. Positive axial acceleration in the core acts to resist diffusion of vorticity, since air-parcels are thereby stretched and so reduced in diameter. Conservation of angular momentum and a reduction in diameter act to concentrate vorticity against radial diffusion.

The height to which a dust devil persists before breakdown to a turbulent plume affects the wind strength at the ground since the plume does not contribute to those flows.

The End Wall Effect

The end wall effect biases the cyclostrophic balance (wherein radial pressure gradient is balanced by centrifugal force) in the vortex flows near the ground. The pressure gradient arises from the buoyancy of the core. Friction at the interface and wind shear above it reduce tangential velocity and centrifugal force, allowing air to be drawn in by the radial pressure gradient. Swirl is thus concentrated into the base of the vortex.

Barcilon [1] made an analysis of the end wall effect. The predicted stream-lines of figure 1 are similar to those seen in figure 2 where the inflow is made visible by ripples on the calm sea.

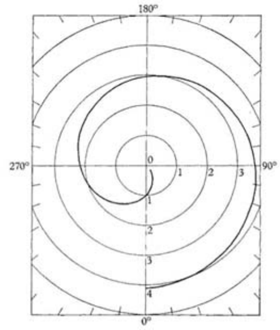

Figure 1. Streamline in plan (Barcilon [1])

Figure 2. Waterspout off the Florida Keys (Renno [10])

More recently numerical analysis using computer models has been possible. For instance Lewellen and Lewellen [5] have used LES (Large Eddy Simulation) models of the flows within vortices. This subject has been intensely studied over a long time because of the safety implications of such corner flows in tornados.

The Heat Engine

In Renno et al. [11] the authors model the thermodynamics of the convective processes in dust devils that maintain the pressure differential in the core. They assume any convective phenomenon is a heat-engine and consider a dust-devil in quasi-steady state, so work done by the heat engine balances mechanical friction, in order to model the maximum bulk-thermodynamic intensity of a vortex in cyclostrophic balance. They assume flows are incompressible, heat input is sensible heat flux at the surface and heat output is radiation from subsiding air-flows (at the average temperature of the convective slab). They assume the convective flows are adiabatic, that the engine is reversible and that energy loss through mixing of high entropy updraft air with lower entropy ambient air is implicitly included through the definition of the cold temperature with respect to CAPE. By following an air parcel through a path as shown in figure 3, they derive relationships for pressure differentials and cyclostrophic flow velocities.

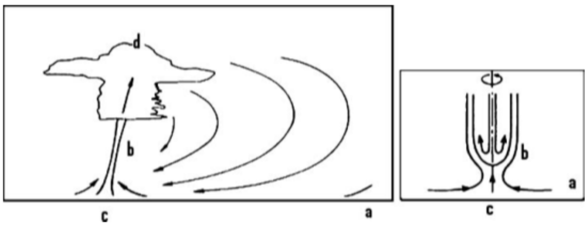

Figure 3. Vortex Circulation (Renno [10])

In Renno and Bluestein [9] the analysis is extended to waterspouts. They derive an expression for the pressure differential arising from core buoyancy:

$$\Delta p \approx p_\infty \left\{1 - \exp\left\{\left(\frac{\gamma\eta}{\gamma\eta-1}\right)\left[\left(\frac{C_p}{R}\right)\left(\frac{T_0-T_\infty}{\overline{T_s}}\right)+\left(\frac{L_v}{R}\right)\left(\frac{r_0-r_\infty}{\overline{T_s}}\right)\right]\right\}\right\} \qquad (1)$$

Cyclostrophic balance is assumed. Tangential velocity at the radius of the core wall using the ideal gas law is then:

$$V_a \approx \sqrt{RT_\infty \left\{1 - \exp\left\{\left(\frac{\gamma\eta}{\gamma\eta-1}\right)\left[\left(\frac{C_p}{R}\right)\left(\frac{T_0-T_\infty}{\overline{T_s}}\right)+\left(\frac{L_v}{R}\right)\left(\frac{r_0-r_\infty}{\overline{T_s}}\right)\right]\right\}\right\}} \qquad (2)$$

- Δp is the pressure differential and p_∞ is the pressure at infinite radius and ground level
- γ is the fraction of frictional energy dissipated at the surface
- $\eta = \left(\frac{T_h - T_c}{T_h}\right)$ is the reversible efficiency of the heat engine
- C_p is heat capacity of air at constant pressure
- R is the gas constant for air
- T is absolute temperature
- L_v is latent heat of vaporisation of water per unit mass
- r is the water vapour mixing ratio
- $\overline{T_s}$ is the entropy averaged temperature of heating
- a is the radius of the core

For dust devils the change in mixing ratio can be neglected (not so for waterspouts). The papers give estimates of tangential wind speeds in the core wall that are well supported with cited observations for dust devils and more generally supported for waterspouts.

Hydrodynamic Stability and Modified Turbulence

The core of the vortex must be constrained to suppress mixing and make energy available to drive flows at the ground. This constraint comes from radial hydrodynamic stability in the core wall arising from two factors: cyclostrophic balance and stable stratification of density.

Under normal flow conditions the high Reynolds number at the core wall of a concentrated vortex would produce rapid turbulent mixing of all quantities, including temperature and vorticity, as seen in a turbulent plume Rouse et al. [12] and Morton et al. [7]. The normal process of turbulent diffusion involves a cascade of scale, which greatly accelerates diffusion.

This cascade is interrupted by sufficient hydrodynamic stability in the core wall.

Lewellen [6] suggested the Richardson number criterion of stratified turbulence can be modified for axisymmetric swirling flows to give a criterion for stability that also includes the gradient of potential temperature.

$$\frac{\theta'}{\theta_0} < \frac{2\Gamma'}{\Gamma} - \frac{w'^2 r^3}{4\Gamma^2} \qquad (3)$$

where θ is potential temperature, Γ is circulation, w is axial velocity, r is radius and the prime denotes differentiation with respect to radius. Wall stability is increased with increasing vorticity and temperature gradient, since the temperature gradient at the wall of a buoyancy vortex is negative.

The question that then arises is how to predict the height to which hydrodynamic stability will allow the vortex to persist before breakdown to a plume. Dergarabedian [3] (Fendell 1967) offers an analysis of vortex intensification and decay based on an asymptotic expansion of the Navier Stokes equations non-dimensionalised with respect to an Ekman number:

$$E = \nu / \Gamma_\infty \qquad (4)$$

where ν is kinematic viscosity and Γ_∞ is the environmental circulation.

The analysis suggests that for a vortex to concentrate the Ekman number must be much less than one and the product of the area of the updraft and the gradient of average vertical velocity within it must be significantly greater than the kinematic viscosity. Therefore a large dust devil requires only a small vertical acceleration for the core to be sustained and concentrated, but smaller vortices require greater vertical acceleration.

Necessarily Divergent Lapse Rates

A theory of necessarily divergent lapse rates is suggested to explain vortex height for tall dust devils, which assumes the core persists and advects upwards while CAPE is positive and the core lapse rate is less than the environmental lapse rate. Positive vertical acceleration arises from the divergent lapse rates as the rate of release of CAPE increases with height. Concentration of vorticity overcomes diffusion of vorticity, so the core advects upwards. So a dust-devil with an adiabatic core lapse rate is sustained while rising through a superadiabatic environmental lapse rate.

Once there is no divergence and no vertical acceleration, the vortex decays to a turbulent plume rapidly, in a height of the order of the diameter from which swirl was concentrated.
Energy is expended in the plume in lifting and warming of entrained air through turbulent mixing. It is therefore suggested that the plume should be considered as the cold reservoir of the heat-engine driving the vortex flows.

Evidence from the Literature

Laboratory Experiments

Mullen and Maxworthy [8] used a vortex-generator with adjustable peripheral vanes to generate swirl and a heated plate to induce buoyancy, mounted in a draft-proof cabinet, lightly extracted from above to produce a neutral stratification. Their analysis is based on functional parameters well established in the analysis of turbulent plumes (Morton et al. [7]) scaled to power input.

Temperature profiles were derived for a range of vane angles and power inputs, as seen for example in figure 4, showing a 2-cell structure.

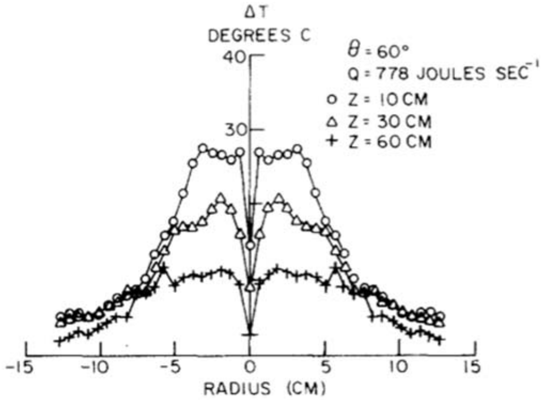

Figure 4. Scaled Temperature excess vs Radius, 60° vane angle, 778W (Mullen and Maxworthy [8])

Figure 5 shows the rate of decay of maximum temperature differential (scaled to input power) with height for two vortices. They show a common inflection point in their scaled temperature profiles. Above the inflection the profiles show a $z^{-5/3}$ dependence, which is characteristic of a turbulent plume. The height of the inflection is approximately equal to the diameter of the swirl vanes.

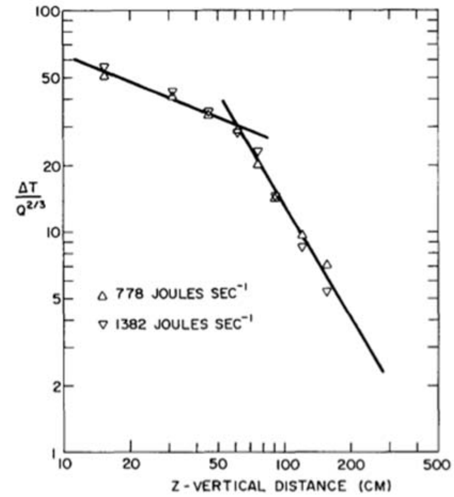

Figure 5. Scaled Temperature Excess vs Height (Mullen and Maxworthy [8])

Table 1 shows results from Mullen and Maxworthy [8] annotated to show vortices for which the temperature profile is given or can be extrapolated from the paper.

Vortex circulation strength and core diameter

Vane angle (°)	346 J sec⁻¹	778 J sec⁻¹	1058 J sec⁻¹	1382 J sec⁻¹
30	Γ = 968 d = 9.6	1 Γ = 1181 d = 11.5	2	Γ = 1452 d = 12.7
45	Γ = 1323 d = 17.2	3 Γ = 1542 d = 18.4	4 Γ = 2142 d = 17.8	Γ = 1910 d = 19.7
60	Γ = 1865 d = 19.9	5 Γ = 2600 d = 20.6	6 Γ = 2994 d = 20.8	7 Γ = 2916 d = 21.6
75	d = 21.8	8 Γ = 3671 d = 26.6	9 Γ = 4342 d = 25.6	Γ = 5445 d = 26.2

Γ in cm^2 sec^{-1} and d in cm.

Table 1. Circulation Strengths, Mullen and Maxworthy [8] annotated

Tangential velocity is estimated at the core wall, taken to have the outer diameter of the area of steep radial temperature gradient at the base of the vortex. For instance, figure 5 shows profiles for vortex 5 in Table 1, $d_t = 16$cm. In Table 1 d is given as the maximum extent of the bubble-tracks within the core and $d_t < d$. The tangential velocity is then calculated as $V_{ot} = \Gamma/\pi d_t$ (5)

Equation (2) is then used to calculate V_a

- The friction efficiency is assumed to be $\gamma = 95\%$
- The thermodynamic efficiency is assumed to be $\eta = \frac{T_h - T_c}{T_h}$
- $T_h = T_\infty + \Delta T_h$ and $T_c = T_\infty + \Delta T_c$
- T_h is taken from the given profiles or extrapolations
- ΔT_c is estimated from figure 9 as being $\Delta T_c = 30 * Q^{2/3}$ °C

Using $\Delta T_c = 0$ would overestimate velocities seen in the experiment, although $\Delta T_c = \Delta T_h/2$ is a closer approximation. Figure 6 shows the correlation obtained.

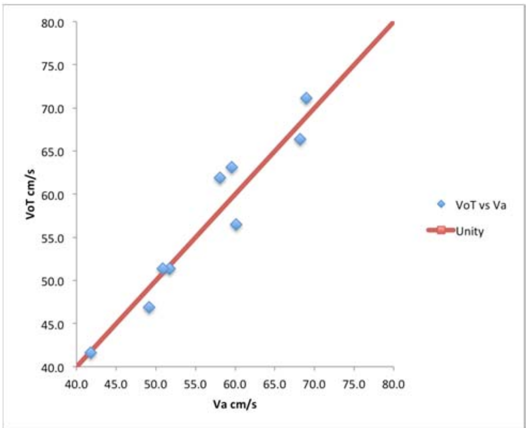

Figure 6. Tangential Velocity – modelled and measured

Field Data for Dust Devils

Ryan and Carroll [13] made a field study in the Mojave Desert collecting atmospheric data to 1500m. Wind velocity is shown as proportional to the square root of the height of the superadiabatic layer with much scatter. This is consistent with the proposed theory. Hess and Spillane [4] made a study of dust-devils occurring in Australia and noted a correspondence in the statistics for dust-devil height, shown in figure 7 and vertical velocity variance normalised by the convection velocity w^* [2,14] - shown in figure 8. Figure 7 shows two populations. The upper population of tall dust devils shows a mean height of 0.51h.

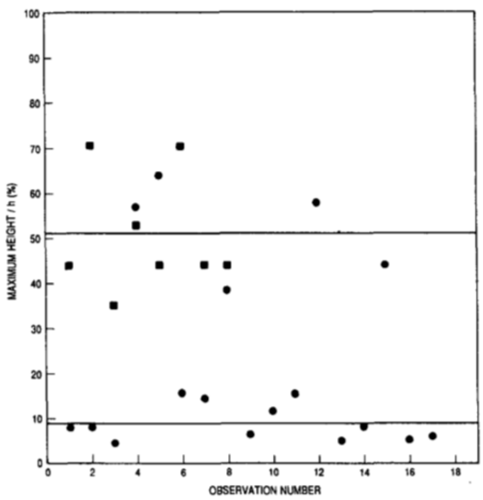

Figure 7. Heights of dust devils under strong convection (Hess and Spillane [4])

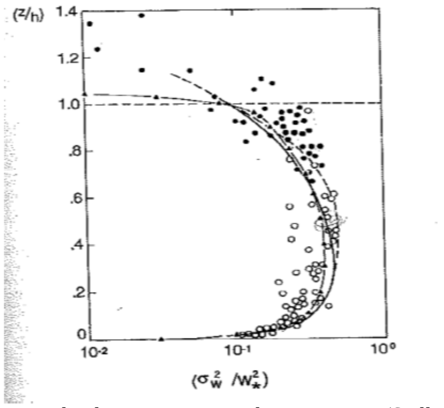

Figure 8. Vertical velocity variance under convection (Spillane and Hess [14])

At the edges of dry deserts deep absolutely unstable superadiabatic boundary layers may develop above the desert floor as a result of advection from adjacent colder zones moving cold air over heating from the ground, allowing tall dust-devils to be driven by dry adiabatic convection.

Figure 9 is a bare tephigram (with the structure omitted for clarity) showing two different convective layers sharing the same height (*h*). The core air ascends following the adiabatic lapse rate shown in red. *Ts* is the surface temperature. *Tconv* is the temperature at the top of the convective layer.

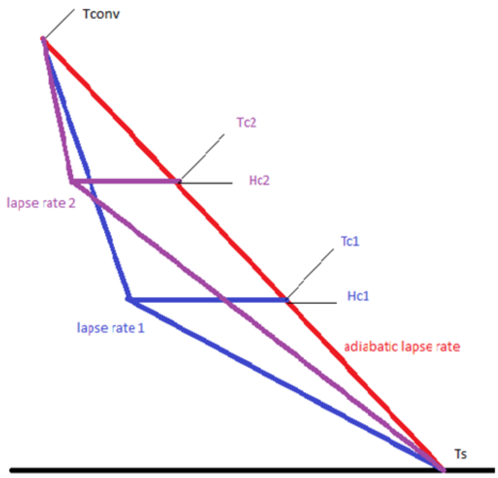

Figure 9. Variation in dust-devil height as a fraction of the convective height

The first atmosphere is shown in blue. The environmental lapse rate is superadiabatic until height *Hc1*. The cold reservoir temperature is then *Tc1*. The second atmosphere is shown in purple. The environmental lapse rate is superadiabatic until *Hc2*. The cold reservoir temperature is then *Tc2*. Linear environmental lapse rates are shown for clarity. In reality they presumably change monotonically but the argument still holds. So $H_{c2} > H_{c1}$ and $T_{c2} < T_{c1}$ so the vortex thermodynamic efficiency is higher under the second atmosphere than under the first and the wind speeds produced are higher.

From figure 9 it is obvious that the mean height of tall dust devils will tend to half the convective height, as shown in figure 7 and the cold reservoir temperature will tend to the average temperature of the convective slab, as per Renno et al [11] but variation around the mean can be expected. The cold reservoir temperature will be higher than the environmental temperature at the height of the plume.

The proposed theory suggests that a positive gradient of vertical velocity is necessary to the formation of tall, powerful dust devils, as well as buoyancy and swirl. This explains why they are rare, even where buoyancy and swirl are readily available, as they will only occur within deep superadiabatic layers.

Conclusions

A theory of necessarily divergent lapse rates is developed to allow the heat engine theory of Renno et al.[11] and Renno and Bluestein [9] to be used to explain wind velocities occurring in laboratory vortices and dust devils and statistics of the height of dust devils, by a modification of the assumptions with respect to the cold reservoir.

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